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Cross-stream migration of non-spherical particles in a second-order fluid – theories of particle dynamics in arbitrary quadratic flows

Cheng-Wei Tai\textsuperscript{1}, Shiyan Wang\textsuperscript{1} and Vivek Narsimhan\textsuperscript{1}$\dagger$

\textsuperscript{1}Davidson School of Chemical Engineering, Purdue University, West Lafayette, IN 47907, USA

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Our recent paper (Tai \textit{et al.} 2020) discussed the polymeric force/torque acting on a non-spherical particle in a second order fluid (co-rotational limit). It contains two sets of typos. We will correct these shortcomings below.

- Equation (2.27) in Tai \textit{et al.} (2020) should read

$$\frac{2\mu}{\psi_1} T_i^p = -\frac{\partial T_i^N}{\partial t} + \varepsilon_{ijk}(u_j^c - U_j)F_k^{(1)} + 2\varepsilon_{ijk}E_{jm}S_{mk} - \varepsilon_{ijk}T_j^N\Omega_k + \frac{3}{2}\varepsilon_{ijk}\Gamma_{j\rho q}^{(3)'} F_{k\rho q}^{(3)'}$$

$$+ \frac{1}{5}\varepsilon_{ijk}\tau_j H_k - \frac{1}{10}\varepsilon_{ijk}\tau_j B_k + \frac{1}{6}\varepsilon_{ijk}\Omega_{jm}\Theta_{mk},$$

(2.27)

where the second term on the right hand side is now $\varepsilon_{ijk}(u_j^c - U_j)F_k^{(1)}$ instead of $\varepsilon_{ijk}u_j^c F_k^{(1)}$ in the original manuscript. Similarly, equation (3.2) should be corrected to (Brunn 1977):

$$T_p^i = -\frac{\psi_1}{2\mu} \left( \frac{\partial T_i^N}{\partial t} + \varepsilon_{ijk}F_j^{(1)}(u_k^c - U_k) + \varepsilon_{ijk}T_j^N\Omega_k + 2\varepsilon_{ijk}E_{km}S_{mj} \right).$$

(3.2)

where the second term in the parentheses is now $\varepsilon_{ijk}F_j^{(1)}(u_k^c - U_k)$ instead of $\varepsilon_{ijk}F_j^{(1)}u_k^c$ as stated before. Equation (3.4) should be corrected to:

$$T_i^{tot} = 8\pi \mu R^3 (\Omega_i - \omega_i) - \frac{\psi_1}{2\mu} \varepsilon_{ijk} \left[ 6\pi \mu R (u_j^c - U_j)(u_k^c - U_k) \right]$$

$$- \frac{\psi_1}{2\mu} \varepsilon_{ijk} \left[ 8\pi \mu R^3 (\Omega_j - \omega_j)\Omega_k \right] + \frac{\psi_1}{\mu} \varepsilon_{ijk}E_{jm}S_{mk} - 4\pi R^3 \psi_1 \frac{\partial}{\partial t}(\Omega_i - \omega_i).$$

(3.4)

For those interested, the origin of the above corrections come from the time derivatives in the polymeric torque integral in equation (2.18). The first term on the right hand side

$\dagger$ Email address for correspondence: vnarsim@purdue.edu
of the equation (2.18) can be written as:

\[
2 \psi_1 F^p_i = \int_{S_\infty} \varepsilon_{ijk} x_j \left( \frac{1}{\mu} \frac{\partial P^N}{\partial t} \delta_{kp} - \frac{\partial \gamma_{kp}}{\partial t} \right) n^\infty_p dS \\
+ \int_{S_\infty} \frac{1}{\mu} \frac{\partial u_m}{\partial x_j} \left( P^N \delta_{ij} - \mu \gamma_{ij} \right) n^\infty_m dS \\
+ \int_{S_\infty} \left( \frac{1}{4} \gamma_{km} \gamma_{mk} \delta_{ij} - \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} + \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right) n^\infty_j dS,
\]

(2.17)

\[
2 \psi_1 T^p_i = \int_{S_\infty} \varepsilon_{ijk} x_j \left( \frac{1}{\mu} \frac{\partial P^N}{\partial t} \delta_{kp} - \frac{\partial \gamma_{kp}}{\partial t} \right) n^\infty_p dS \\
+ \int_{S_\infty} \varepsilon_{ijk} x_j \frac{\partial u_m}{\partial x_p} \left( \frac{1}{\mu} P^N \delta_{kp} - \gamma_{kp} \right) n^\infty_m dS \\
- \int_{S_\infty} \varepsilon_{imk} \left( \frac{1}{\mu} P^N \delta_{kp} - \gamma_{kp} \right) u_m n^\infty_p dS \\
+ \int_{S_\infty} \varepsilon_{ijk} x_j \left( \frac{1}{4} \gamma_{mn} \gamma_{mn} \delta_{kp} - \frac{\partial u_m}{\partial x_k} \frac{\partial u_m}{\partial x_p} + \frac{\partial u_k}{\partial x_m} \frac{\partial u_p}{\partial x_m} \right) n^\infty_p dS.
\]

(2.18)

The corrections will modify one plot in the manuscript, which is figure 3(c) in section 3.2.2. The nondimensionalized polymeric torque is now on the order of \(O(10^{-3})\), which is within the error of the BEM simulation. All other results in the paper are unaffected, since the corrected term is proportional to the Newtonian force \(F^{(1)}_i\), and all the other scenarios examined have \(F^{(1)}_i = 0\) to leading order.

- We noticed two typographical errors in equations (2.17) and (2.18), where a plus sign is missing between the first and second integral. The equations should be corrected to:

\[
2 \psi_1 F^p_i = \int_{S_\infty} \varepsilon_{ijk} x_j \left( \frac{1}{\mu} \frac{\partial P^N}{\partial t} \delta_{kp} - \frac{\partial \gamma_{kp}}{\partial t} \right) n^\infty_p dS \\
+ \int_{S_\infty} \frac{1}{\mu} \frac{\partial u_m}{\partial x_j} \left( P^N \delta_{ij} - \mu \gamma_{ij} \right) n^\infty_m dS \\
- \int_{S_\infty} \varepsilon_{imk} \left( \frac{1}{\mu} P^N \delta_{kp} - \gamma_{kp} \right) u_m n^\infty_p dS \\
+ \int_{S_\infty} \varepsilon_{ijk} x_j \left( \frac{1}{4} \gamma_{mn} \gamma_{mn} \delta_{kp} - \frac{\partial u_m}{\partial x_k} \frac{\partial u_m}{\partial x_p} + \frac{\partial u_k}{\partial x_m} \frac{\partial u_p}{\partial x_m} \right) n^\infty_p dS.
\]

(2.18)

REFERENCES


FIGURE 3. (c) Non-dimensionalized polymeric torque in the z-direction for different values of $y_0$. The parameters are $L = 10$, $R = 1$, $Wi = 0.1$. $Wi$ is the Weissenberg number, defined by $Wi = \psi_1 u^m / L\mu$. 

$2\mu u^m \frac{\partial \psi_1}{\partial y_0}$